DECROLY EDUCATION CENTRE

Logarithmic Audio Problem Set

Logarithms in Sound Engineering: Decibels, Pitch Scales, and Digital Production

20 questions • Increasing difficulty • Real-world scenarios
For use with: Physics exams (A-Level, AP, IB), audio engineering courses, and self-study.

Section 1: Decibel Basics (Questions 1–5)

Q1 – Threshold of Hearing

The reference sound intensity $I_0 = 1 \times 10^{-12}\ \mathrm{W/m^2}$.
A whisper produces an intensity of $1 \times 10^{-10}\ \mathrm{W/m^2}$.
Calculate the sound intensity level in decibels (dB).

Q2 – Doubling Intensity

A speaker outputs 0.01 W/m^2. You double the electrical power, increasing intensity to 0.02 W/m^2.
How many dB increase does this produce?

Q3 – Real-world scenario: Studio Monitor Calibration

A studio monitor produces 85 dB at 1 meter.
If you move to 2 meters (inverse square law: intensity $\propto 1/r^2$), what is the new dB level?
(Hint: First find the intensity ratio, then convert to dB change.)

Q4 – Hearing Safety

Prolonged exposure above 85 dB can cause hearing damage.
A rock concert measures 110 dB at the audience.
How many times more intense is this than the 85 dB safety threshold?

Q5 – Multiple Sources

Two identical loudspeakers each produce 80 dB at a point when played alone.
If both play simultaneously (coherently, same phase), what is the total dB?
(Assume constructive interference doubles the pressure amplitude, not intensity.)

Section 2: Pitch, Frequency, and Octaves (Questions 6–10)

APPROVED
10/06/2026

Q6 – Octave Ratio

The frequency of middle C is 261.63 Hz.
What is the frequency of the C one octave higher?

Q7 – Equal Temperament Semitone

In 12-tone equal temperament, each semitone has a frequency ratio of $2^{1/12} \approx 1.05946$.
Starting from A4 = 440 Hz, calculate the frequency of B4 (2 semitones higher).

Q8 – Cents between two frequencies

Calculate the number of cents between 440 Hz and 445 Hz.
Formula: $\text{cents} = 1200 \times \log_2\left(\frac{f_1}{f_2}\right)$.

Q9 – Real-world scenario: Tuning a Guitar

A guitar string should be 110 Hz (A2). It is actually 112 Hz.
How many cents sharp is it?
Is this within the typical ±5 cent tolerance for professional tuning?

Q10 – Frequency from cents deviation

A synth pad is tuned to 440 Hz. The producer detunes it by +50 cents.
What is the new frequency?

Section 3: Intermediate – Combining dB with Ratios (Questions 11–15)

Q11 – Sound Pressure Level (SPL) formula

SPL in dB is often defined as $20 \log_{10}(p/p_0)$ where p is sound pressure.
If a measurement gives 94 dB SPL, what is the pressure ratio p/p_0?

Q12 – Real-world scenario: Fan Noise Reduction

A server room fan produces 70 dB. After acoustic treatment, the sound pressure drops by a factor of 4 (p_new = p_old / 4).
What is the new dB level?

APPROVED
10/06/2026

Q13 – Adding incoherent sound sources

Two independent noise sources produce 60 dB and 65 dB.
What is the total dB?
(Incoherent addition: total intensity = sum of intensities.)

Q14 – Signal-to-Noise Ratio (SNR) in audio

A recording has signal level 78 dB and noise floor 42 dB.
What is the SNR in dB?
Then, if the signal is amplified by +20 dB, what is the new SNR?

Q15 – Real-world scenario: Live sound system

A PA system produces 100 dB at 10 meters. The sound engineer wants 105 dB at that same position.
By what factor must the acoustic power increase?
(Hint: dB change = 10 $\log_{10}$(P_new/P_old).)

Section 4: Advanced – Logarithms in Filters & Digital Audio (Questions 16–20)

Q16 – Decade and octave slopes

A low-pass filter attenuates signals at 12 dB per octave.
How many dB attenuation does it provide per decade (10× frequency change)?
(Recall: 1 decade = $\log_2(10) \approx 3.322$ octaves.)

Q17 – Real-world scenario: Equal-loudness contour (Fletcher-Munson)

At 100 Hz, a sound must be 30 dB SPL to sound as loud as a 1 kHz tone at 20 dB SPL.
What is the dB difference in physical intensity?
Then, express the required intensity ratio as a power of 10.

Q18 – Digital audio dynamic range (bit depth)

Dynamic range in dB ≈ $6.02 \times N$ for an N-bit digital system (dither ignored).
A 16-bit CD has ~96 dB range. A 24-bit studio recording has ~144 dB range.
How many times greater is the amplitude resolution of 24-bit compared to 16-bit?
(Amplitude ratio = 2^(N_diff).)

Q19 – Real-world scenario: Recording a quiet passage

A quiet violin passage has peak intensity 1×10^{-8} W/m². The room noise is 35 dB.
Will the violin be audible above the noise floor?
(Calculate violin dB first, using $I_0 = 1 \times 10^{-12}$ W/m².)

Q20 – Master problem: Studio combination

A studio has three sound sources:

- Source A: 82 dB
- Source B: 78 dB
- Source C: 85 dB
  All are incoherent (e.g., different instruments).

a) Calculate the total dB.
b) If the engineer reduces Source C by 6 dB, what is the new total?
c) The target level for mastering is 89 dB. How many dB of additional gain is needed from the master bus?

Answer Key

Section 1

Q1
$\text{dB} = 10\log_{10}(I/I_0) = 10\log_{10}(1 \times 10^{-10}/1 \times 10^{-12}) = 10\log_{10}(100) = 10 \times 2 = 20\,\text{dB}$

Q2
Intensity ratio = 2.
$\Delta\text{dB} = 10\log_{10}(2) \approx 10 \times 0.3010 = 3.01\,\text{dB}$

Q3
Distance doubles → intensity drops by factor of 4 (since $1/r^2$).
$\Delta\text{dB} = 10\log_{10}(1/4) = 10 \times (-0.6021) = -6.02\,\text{dB}$
New level = 85 – 6.02 = 78.98 dB ≈ 79 dB.

Q4
Difference = 110 – 85 = 25 dB.
$25 = 10\log_{10}(I_{\text{concert}}/I_{\text{threshold}}) \rightarrow \log_{10}(\text{ratio}) = 2.5 \rightarrow$ ratio = $10^{2.5} \approx 316.2$ times more intense.

Q5
Important: Doubling pressure amplitude → intensity ×4.
Each speaker alone: I_1 corresponds to 80 dB.
Two together: I_total = 4 × I_1.
$\text{dB total} = 80 + 10\log_{10}(4) = 80 + 6.02 = 86.02\,\text{dB}.$

APPROVED
10/06/2026

Section 2

Q6

One octave higher: multiply by 2.

$261.63 \times 2 = 523.26\,\text{Hz}$.

Q7

Two semitones: factor = $(2^{1/12})^2 = 2^{2/12} = 2^{1/6} \approx 1.12246$.

$440 \times 1.12246 \approx 493.88\,\text{Hz}$ (which is B4).

Q8

$\text{cents} = 1200\log_2(445/440)$

$445/440 = 1.011364$

$\log_2(1.011364) = \ln(1.011364)/\ln(2) \approx 0.01130/0.693147 \approx 0.01630$

Cents = $1200 \times 0.01630 \approx 19.56$ cents (about 20 cents sharp).

Q9

$\text{cents} = 1200\log_2(112/110) = 1200\log_2(1.018182)$

$\ln(1.018182) \approx 0.018018$, divide by 0.693147 → 0.02599

Cents ≈ $1200 \times 0.02599 \approx 31.2$ cents sharp.

Not within ±5 cents – needs tuning.

Q10

$50 = 1200\log_2(f/440)$

$\log_2(f/440) = 50/1200 = 0.0416667$

$f/440 = 2^{0.0416667} \approx e^{0.0416667\times\ln 2} = e^{0.02888} \approx 1.0293$

$f \approx 440 \times 1.0293 = 452.89\,\text{Hz}$.

Section 3

Q11

$94 = 20\log_{10}(p/p_0) \rightarrow \log_{10}(p/p_0) = 94/20 = 4.7$

$p/p_0 = 10^{4.7} \approx 5.012 \times 10^4$.

Q12

Pressure factor = 1/4.

$\Delta\text{dB} = 20\log_{10}(1/4) = 20 \times (-0.6021) = -12.04\,\text{dB}$

New level = 70 – 12.04 = 57.96 dB ≈ 58 dB.

Q13

Convert each dB to intensity ratio (relative to I_0):

I_A = $10^{60/10} = 10^6$ (in units of I_0)

I_B = $10^{65/10} = 10^{6.5} \approx 3.162 \times 10^6$

Total I = $1 \times 10^6 + 3.162 \times 10^6 = 4.162 \times 10^6$

APPROVED
10/06/2026

dB total = $10\log_{10}(4.162 \times 10^6) = 10(6 + \log_{10} 4.162)$
$\log_{10} 4.162 \approx 0.619$, so total dB = 10 × 6.619 = 66.19 dB.

Q14
SNR = signal dB – noise dB = 78 – 42 = 36 dB.
After amplifying both signal and noise equally (+20 dB), SNR remains 36 dB (both increase, ratio unchanged).

Q15
ΔdB = 5 = $10\log_{10}(P_{\text{new}}/P_{\text{old}})$
$\log_{10}(\text{ratio}) = 0.5$ → ratio = $10^{0.5} \approx 3.162$ times power.

Section 4

Q16
1 decade = $\log_2(10) \approx 3.322$ octaves.
Attenuation = 12 dB/oct × 3.322 oct ≈ 39.86 dB per decade.

Q17
Physical dB difference = 30 dB SPL – 20 dB SPL = 10 dB.
Intensity ratio = $10^{10/10} = 10^1 = 10$ times more intense at 100 Hz.

Q18
Bit depth difference = 24 – 16 = 8 bits.
Amplitude resolution ratio = $2^8 = 256$ times more amplitude steps.

Q19
Violin dB = $10\log_{10}(1 \times 10^{-8}/1 \times 10^{-12}) = 10\log_{10}(10^4) = 40\,\text{dB}$.
Noise = 35 dB.
Violin is 5 dB above noise – audible, but quiet. (In practice, masking may apply.)

Q20a
Convert to intensity ratios:
A: $10^{8.2} = 1.585 \times 10^8$
B: $10^{7.8} = 6.310 \times 10^7$
C: $10^{8.5} = 3.162 \times 10^8$
Sum = $(1.585 + 0.631 + 3.162) \times 10^8 = 5.378 \times 10^8$
Total dB = $10\log_{10}(5.378 \times 10^8) = 10(8 + \log_{10} 5.378)$
$\log_{10} 5.378 \approx 0.731$ → total = 10 × 8.731 = 87.31 dB.

Q20b
Reduce C by 6 dB → new C level = 85 – 6 = 79 dB.
C intensity = $10^{7.9} = 7.943 \times 10^7$
Sum = A + B + C_new = $1.585 \times 10^8 + 0.631 \times 10^8 + 0.794 \times 10^8 = 3.010 \times 10^8$
dB new = $10\log_{10}(3.010 \times 10^8) = 10(8 + \log_{10} 3.010)$
$\log_{10} 3.010 \approx 0.479$ → total = 84.79 dB.

Q20c
Desired = 89 dB, current (from a) = 87.31 dB.
Gain needed = 89 – 87.31 = 1.69 dB.

Bonus Real-world Extensions (For Discussion)

1. **Why does a 3 dB increase require doubling the power, but a 10 dB increase sounds twice as loud?**
   (Answer: Loudness perception is logarithmic; 10 dB ≈ perceived doubling.)
2. **In a live venue, why does moving 1 m away from a speaker sometimes give more than a 6 dB drop?**
   (Answer: Reflections and room modes can create interference; inverse square assumes free field.)
3. **Why do audio compressors use ratios like 4:1 on a logarithmic dB scale?**
   (Answer: A 4 dB input increase above threshold yields only 1 dB output increase – linear in log domain.)

End of Problem Set

Created for use with "Logarithms in Sound Engineering" explainer video.