Decroly Education Centre

Wave Interference Problem Set

Calculus of Harmony: Constructive & Destructive Interference in Audio Engineering

20 questions • Diagram-based • Phase calculations • Hall simulation mini-cases
For use with: Physics exams (A-Level, AP, IB), acoustic design courses, and studio engineering.

How to use the diagrams

Each question refers to a simple figure. **Imagine or sketch** the diagram as described. For digital use, blank space is provided for your calculations.

Section 1: Basic Two-Source Interference (Questions 1–5)

Q1 – Diagram 1: Two speakers in phase

Diagram description: Two speakers (S1, S2) are 2.0 m apart. A listener stands 3.0 m directly in front of S1. Frequency = 343 Hz. Speed of sound = 343 m/s.

a) Calculate the wavelength.
b) Calculate the path difference from S1 and S2 to the listener.
c) Is the interference constructive, destructive, or intermediate?

Q2 – Diagram 2: Path difference = λ/2

Diagram description: Same setup as Q1, but the listener moves to a position where the path difference is exactly half a wavelength for a 686 Hz tone.
What is the path difference in meters? What type of interference occurs?

Q3 – Phase shift from path difference

Formula: $\Delta\phi = 2\pi\frac{\Delta x}{\lambda}$.
A path difference of 0.25 m for a 500 Hz tone (v = 340 m/s). Calculate Δφ in radians and degrees.

Q4 – Real-world scenario: PA speaker alignment

A subwoofer and a top speaker are 1.0 m apart vertically. The crossover frequency is 100 Hz (λ = 3.4 m at 340 m/s). A listener is exactly midway between them in the far field.
What is the path difference?

Is the interference at the crossover frequency constructive or destructive?
What could the engineer do to fix it?

Q5 – Diagram 3: Two speakers, opposite phase

Diagram description: Two speakers are 1.5 m apart. They are wired out of phase ($\Delta\varphi = \pi$). Frequency = 170 Hz (λ = 2.0 m). A listener is 4.0 m from S1 and 4.5 m from S2.
Calculate the total phase difference (from both path and initial offset). Constructive or destructive?

Section 2: Multiple-Slit & Array Interference (Questions 6–10)

Q6 – Diagram 4: Three speakers in a line

Diagram description: Three identical speakers spaced 0.5 m apart, all in phase. Frequency = 680 Hz (λ = 0.5 m). A listener is very far away, perpendicular to the line (angle 0°).
What is the relative amplitude compared to a single speaker?
(Hint: Phasor addition.)

Q7 – Same array, angle changed

For the array in Q6, at what angle θ (from the perpendicular) does the first destructive interference occur?
(Use $d\sin\theta = \lambda/2$ for two adjacent speakers? But for three, first null happens when the phasors spread 120° apart – but simpler: treat as two-slit pattern for the first minimum.)
Better: Compute the angle where the path difference between adjacent speakers = λ/3 (cancellation for three equally spaced). Accept any correct reasoning.

Q8 – Real-world scenario: Line array in a concert hall

A line array of 8 speakers is used. The vertical spacing is 0.2 m. At 1000 Hz (λ = 0.34 m), what is the vertical angle of the first cancellation lobe?
Use $\theta = \arcsin(\lambda/(N \cdot d))$ for first null of an N-element array? Actually, first null of an array of N sources spaced d: $\sin\theta = \lambda/(Nd)$ for broadside.
Compute that.

Q9 – Diagram 5: Two speakers, listener off-axis

Diagram description: Two speakers 3.0 m apart. Frequency = 172 Hz (λ = 2.0 m). Listener at 5.0 m from the midpoint, offset by 2.0 m sideways.
Calculate exact path difference. Determine interference type.

Q10 – Phase cancellation in stereo recording

Two microphones spaced 0.5 m record a 340 Hz tone from a source 2.0 m away, perpendicular to the mic array axis. The sound arrives at the farther mic with a delay.
Calculate the phase difference. Does this cause comb filtering?

Section 3: Room & Hall Simulation Mini-Cases (Questions 11–15)

Q11 – Mini-case 1: Rectangular room mode

A room has length 8.0 m. Sound speed 340 m/s. The fundamental axial mode (standing wave between two parallel walls) occurs when L = λ/2.
What is the fundamental frequency?
At which frequency does the first destructive interference occur for a listener at the center of the room?

Q12 – Mini-case 2: Concert hall – stage to first row

A violinist is 5.0 m from a reflective back wall. A direct sound and a reflected sound (off the wall) reach an audience member 2.0 m in front of the violinist. The reflection travels 5.0 m to the wall and then back to the listener (total path = 5 + 5 + 2 = 12 m). Direct path = 2.0 m. Path difference = 10 m.
For what frequencies (fundamental and first two harmonics) will there be destructive interference?
Assume λ = 2 × path difference for cancellation (Δx = λ/2, 3λ/2, 5λ/2…).

Q13 – Mini-case 3: Diffraction around a pillar

A sound source is behind a circular pillar of diameter 1.0 m. The frequency is 340 Hz (λ = 1.0 m).
The pillar diameter equals λ.
Does the sound diffract significantly?
What does this imply for audience members in the shadow zone?

Q14 – Mini-case 4: Stage monitor placement

A vocalist uses a floor monitor 1.2 m away. The monitor's reflection from the back wall arrives 0.004 seconds later (path difference ≈ 1.36 m).
Calculate the frequencies that will experience destructive interference (comb filtering).
Use $f = \frac{n \cdot v}{2 \cdot \Delta x}$ for cancellation? Actually, destructive when Δx = (n + ½)λ, so $f = (n + 0.5)v/\Delta x$.
List the first three cancellation frequencies.

Q15 – Mini-case 5: Recording booth – eliminating standing waves

A small booth is 2.5 m wide. A speaker at one wall produces a 68 Hz tone (λ = 5.0 m).
Is this frequency problematic (i.e., does it create a standing wave)?
What is the next higher frequency that will cause a strong axial mode?

Section 4: Advanced – Phase Calculations & Complex Amplitudes (Questions 16–20)

Q16 – Phasor addition: Two waves with phase difference

Wave 1: amplitude A, phase 0. Wave 2: amplitude A, phase 60°.
What is the resultant amplitude in terms of A?
(Hint: $A_{\text{total}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi}$.)

Q17 – Three-wave interference with unequal amplitudes

Three sources: A1 = 1.0, A2 = 0.8, A3 = 0.6. Phases: 0°, 90°, 180°.
Calculate the resultant amplitude and phase.

Q18 – Real-world scenario: Active noise cancellation

A noise-cancelling headphone measures an incoming wave with amplitude 1.0 Pa at the microphone. It generates an anti-noise wave with amplitude 0.95 Pa but exactly opposite phase (180°).
What is the residual amplitude?
What is the noise reduction in dB?
(Use pressure ratio, then $20\log_{10}(p_{\text{res}}/p_{\text{in}})$.)

Q19 – Hall simulation: Combining direct and reflected sound

In a hall, the direct sound arrives with pressure amplitude 1.0. A single strong reflection arrives 3 ms later with amplitude 0.6. Frequency = 500 Hz (T = 0.002 s).
Calculate the phase difference due to the 3 ms delay.
Find the total pressure amplitude.
Does this cause constructive or destructive interference at that frequency?

Q20 – Master problem: Three-source array for beam steering

Three speakers in a line, spacing d = 0.25 m. Desired to steer the main lobe to 30° from perpendicular.
What time delay (in ms) should be applied to each successive speaker to achieve this?
Assume speed 343 m/s.
Formula: delay = $(d\sin\theta)/v$. Compute for adjacent speakers.
Then, at the steered angle, is the interference fully constructive? Why?

Answer Key

Section 1

Q1
a) $\lambda = v/f = 343/343 = 1.0$ m.
b) Path to S1 = 3.0 m. Path to S2: distance = $\sqrt{(2.0^2 + 3.0^2)} = \sqrt{13} \approx 3.606$ m. $\Delta x = 3.606 - 3.0 = 0.606$ m.
c) $\Delta x/\lambda = 0.606$ → not an integer or half-integer. Intermediate, closer to constructive ($0.606\lambda \approx 0.6\lambda$, phase ~ $0.6\times360° = 216°$, slightly destructive? Actually 0.6λ gives 216° phase, which is between 180° and 360°: moderate destructive. But standard answer: intermediate.

Q2
$\Delta x = \lambda/2 = (v/f)/2 = (343/686)/2 = (0.5)/2$? Wait: f=686, $\lambda=343/686=0.5$ m. Half λ = 0.25 m. Destructive.

Q3
$\lambda = v/f = 340/500 = 0.68$ m. $\Delta\varphi = 2\pi\ (0.25/0.68) = 2\pi \times 0.36765 = 2.31$ rad. In degrees: $2.31 \times (180/\pi) \approx 132.3°$.

Q4
Far field, midway ⇒ path difference ≈ 0 (symmetry). $\Delta\varphi = 0$ → constructive. To fix, delay one speaker or invert phase.

Q5
Path difference = 4.5 – 4.0 = 0.5 m. $\lambda = v/f = 340/170 = 2.0$ m. $\Delta\varphi_{path} = 2\pi\ (0.5/2.0) = \pi/2$ (90°). Initial offset = π (180°). Total $\Delta\varphi = \pi/2 + \pi = 3\pi/2$ (270°). cos(270°)=0 → partial? Actually $\cos(3\pi/2)=0$, amplitude = $A\sqrt{(1+1+2\cos\Delta\varphi)} = A\sqrt{2}$? Wait, cos270=0, so amplitude = $A\sqrt{2} \approx 1.414A$. Not perfectly constructive or destructive – intermediate, closer to constructive? 270° is same as -90°, gives amplitude $A\sqrt{2}$. So intermediate.

Section 2

Q6
$\lambda = v/f = 340/680 = 0.5$ m. Spacing = λ, so all in phase at 0°. Relative amplitude = 3A. (Constructive)

Q7
For three sources equally spaced, first null occurs when phasors sum to zero. For three equal amplitudes, phases must be 0°, 120°, 240° relative. Path difference between adjacent = $\lambda/3$. So $d \sin\theta = \lambda/3$. d=0.5, λ=0.5 → $\sin\theta = (0.5/3)/0.5 = 1/3 \approx 0.333$, $\theta \approx 19.5°$.

Q8
First null angle for N sources broadside: $\sin\theta = \lambda/(N\cdot d)$ for approximate null? Actually standard array factor null when $\sin\theta = \lambda/(N\ d)$ for N even? Simpler: use $\sin\theta = \lambda/(N\ d)$ gives $\theta = \arcsin(0.34/(8\times0.2)) = \arcsin(0.34/1.6) = \arcsin(0.2125) \approx 12.3°$.

Q9

Coordinates: S1 at (-1.5, 0), S2 at (1.5, 0). Listener at (2.0, 5.0) [since offset 2 m sideways from midpoint]. Distances: $d1 = \sqrt{(2+1.5)^2 + 5^2} = \sqrt{3.5^2+25}=\sqrt{12.25+25}=\sqrt{37.25}\approx 6.10$ m. $d2 = \sqrt{(2-1.5)^2+5^2}=\sqrt{0.5^2+25}=\sqrt{0.25+25}=\sqrt{25.25}\approx 5.025$ m. $\Delta x = 6.10 - 5.025 = 1.075$ m. $\lambda=2.0$ m. $\Delta x/\lambda = 0.5375$. Phase = $0.5375\times 360° \approx 193.5°$ → destructive (near 180°). Yes, destructive.

Q10

Path difference = 0.5 m (microphone spacing). $\lambda = v/f = 340/340 = 1.0$ m. $\Delta\varphi = 2\pi(0.5/1.0)= \pi$ (180°). Complete cancellation at that frequency. Comb filtering occurs.

Section 3

Q11

Fundamental: $L = \lambda/2 \rightarrow \lambda = 2L = 16$ m. $f = v/\lambda = 340/16 = 21.25$ Hz. At center, pressure antinode? For fundamental, center is antinode, so interference is constructive. First destructive occurs at second mode? Actually for listener at center of room length, fundamental gives peak; first null for that listener occurs when $L = \lambda$ (second mode) but center is node? Need to clarify: For open-closed? But room modes are pressure nodes at walls. For length L, frequencies: $f = n\, v/(2L)$, n=1,2,3... At center, n even gives pressure node (destructive), n odd gives antinode (constructive). So first destructive at n=2, $f = 2\times 340/(2\times 8)=340/8=42.5$ Hz.

Q12

$\Delta x = 10$ m. Destructive when $\Delta x = (m+½)\lambda$, m=0,1,2... So $\lambda = 2\Delta x/(2m+1)$. $f = v/\lambda = v(2m+1)/(2\Delta x) = 340(2m+1)/(20) = 17(2m+1)$ Hz. m=0: 17 Hz, m=1: 51 Hz, m=2: 85 Hz. (Very low frequencies, but plausible.)

Q13

Diameter = λ, so significant diffraction (Fresnel number ≈1). Audience in shadow zone will hear some sound; not a complete acoustic shadow.

Q14

$\Delta x = 1.36$ m. Destructive when $\Delta x = (n+½)\lambda \rightarrow \lambda = 2\Delta x/(2n+1)$. $f = v/\lambda = v(2n+1)/(2\Delta x) = 340(2n+1)/(2.72) \approx 125(2n+1)$ Hz. n=0: 125 Hz, n=1: 375 Hz, n=2: 625 Hz.

Q15

$\lambda = v/f = 340/68 = 5.0$ m. Booth width 2.5 m = $\lambda/2$ → fundamental mode exists ($L = \lambda/2$) → problematic (standing wave). Next higher mode: $L = \lambda$ ($f = v/L = 340/2.5 = 136$ Hz) or $L = 3\lambda/2$ ($f = 3v/(2L) = 204$ Hz). Typically next axial mode is 2× fundamental = 136 Hz.

Section 4

Q16

$A_{total} = \sqrt{A^2 + A^2 + 2A^2 \cos 60°} = \sqrt{2A^2 + 2A^2\times 0.5} = \sqrt{2A^2 + A^2} = \sqrt{3A^2} = A\sqrt{3} \approx 1.732A$.

Q17

Convert to complex: $1\angle 0° = 1+0j$. $0.8\angle 90° = 0+0.8j$. $0.6\angle 180° = -0.6+0j$. Sum real: $1 - 0.6 = 0.4$.

Imag: 0+0.8 = 0.8. Resultant = $\sqrt{0.4^2+0.8^2}=\sqrt{0.16+0.64}=\sqrt{0.80}=0.8944$. Phase = atan2(0.8,0.4)= atan(2)=63.4°.

Q18
Residual amplitude = |1.0 – 0.95| = 0.05 Pa. Reduction = 20 $\log_{10}(0.05/1.0)$ = 20 × (-1.301) = –26.0 dB. (Excellent cancellation.)

Q19
Delay Δt = 0.003 s. Phase difference = 2π f Δt = 2π×500×0.003 = 2π×1.5 = 3π rad (equivalent to π rad, since 2π repeats). So Δφ = π (180°). Amplitudes: 1.0 and 0.6, opposite phase → total = |1.0 – 0.6| = 0.4. Destructive interference.

Q20
Delay for adjacent speakers: Δt = (d sinθ)/v = (0.25 × sin30°)/343 = (0.25×0.5)/343 = 0.125/343 ≈ 0.000364 s = 0.364 ms. Yes, at that angle, all waves arrive in phase because the applied delay compensates the path difference. So fully constructive.

End of Problem Set

Created for use with "Calculus of Harmony" explainer video.